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THE DEVELOPMENT OF CALCULUS: A HISTORICAL ANALYSIS OF THE NEWTON - LEIBNIZ CONTROVERSY.

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Abstract

Within the scope of this essay, we will provide an overview of the evolution of calculus within the context of the Indian mathematical heritage. There is a natural division between the two sections of the article. Beginning with the work of stretching up to the work of the Kerala School beginning with, which has a more direct influence on calculus, we will analyse the developments that occurred during what may be considered the classical period in the first part of this article. The second part of this article will be devoted to the discussion of the developments that occurred during this time. In this section, we will talk about some of the contributions that the Kerala School made between the years 1350 and 1500, as described in the fundamental Malayalam work.

Keywords: *Analysis of the Newton, Calculus, and Calculus.*

INTRODUCTION

Carl Boyer, in his groundbreaking history of calculus, which was written sixty years ago, completely disregarded the contributions that Indians made to the conceptual development of the discipline.1. Approximately at the same time as (i) Rama Varma was writing his historical review, Boyer produced his work. In the first edition of the mathematics section of the foundational text Gan, Maru Thampuran and Akhileswarayyar were the ones who carried out the work. After the initial notice by Charles Whish in the early nineteenth century, it appears that the significance of the results and techniques outlined in Yuktha's A (and the work of the Kerala School of Mathematics in general) was forgotten. However, C.T. Rajagopal and his collaborators, in a series of pioneering studies, drew attention to the significance of these results and techniques. As a result of these and later research, a somewhat different perspective of the Indian contribution to the development of calculus has emerged. This may be inferred from the following quotation, which is taken from a modern study on the history of mathematics:

A excellent example of two traditions whose goals were radically different is presented here for our consideration. Despite the fact that there is a chance that the Greek tables of 'trigonometric functions' had been transmitted and modified, the Euclidean philosophy of evidence, which was so prominent in the Islamic world, did not appear to have any impact in India. It would be a question of guesswork to assume that the series was derived from some form of "calculus," as this would call for the assumption to be made.

The one and only exception to this generalisation is a lengthy book that is highly regarded in Kerala and is called as this. This work has something that is more akin to proofs; but, given the distinct paradigm, we should be wary about presuming that they are intended to fulfil the same tasks. However, it does include explanations of how the equations are arrived at, which may be interpreted as a version of the calculus. Although it is difficult to confirm the authorship of this book and the period it was published (the date that is typically assumed to be the sixteenth century), it does provide these explanations.

In point of fact, the Malayalam work Gan. provides a summary of the work that was done by the Kerala School of mathematicians between the years 1350 and 1500 CE. The Kerala School was established by the individual who was a follower of his son D¨amodara and the pupil of the latter. Despite the fact that the accomplishments of the Kerala School are truly remarkable, it has recently come to the attention of the general public that these achievements are in reality a continuation of the previous work done by Indian mathematicians, particularly those of the Aryabhata school, over the time period of 500–1350 CE.

In this first section of the essay, we will discuss a few of the concepts and approaches that were established in Indian mathematics between the years 500 and 1350. These ideas and approaches had an impact on the work that was done by the Kerala School in subsequent years. The following are some of the topics that will be the primary focus of our attention: mathematics of zero and infinity; iterative approximations for irrational numbers; summation (and repeated summation) of powers of natural numbers; the utilisation of second-order differences and interpolation in the calculation of the emergence of the concept of instantaneous velocity of a planet in astronomy; and the calculation of the surface area and volume of a sphere.

No. India was not the country that originated calculus. However, Indian astronomers got extremely near to developing what we now refer to as calculus two hundred years before Newton Leibniz introduced the concept. At some point prior to the year 1500, they had reached a level of development where they were able to apply concepts from integral and differential calculus in order to calculate the infinite series expansions of the sine, cosine, and arctangent function:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} - \cdots,$$
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} - \cdots,$$
$$\arctan x = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} - \cdots.$$

These infinite summations have been presented in a very clear and concise manner by Katz. The sine and cosine expansions were achieved by astronomers through a sequence of issues and insights that ultimately led to the development of these series. I will present a slightly different version now.

Calculus is traditionally presented as a Colle that solves fundamentally geometric issues, such as calculation tangents. This narrative sheds light on calculus by demonstrating that this is not the case. In India, this was not the situation at all. Ideas and answers to issues that are fundamentally algebraic can be found there:

Before the year 1500, India had a deep understanding of geometry. It was a bit player at best, but it was a bit player. As a cautionary tale, the story of calculus in In arise in the absence of the typical geometric context, since what did develop was sterile. This served as the foundation for the story. The did not lead anywhere. In the end, they were forgotten, but academics were able to rescue them.

OBJECTIVES

1. To do research on the evolution of calculus over time.
2. To do research on the historical analysis of the Newton machine.

Greek origins of trigonometry

Trigonometry originated from the study of astronomy and astrology, and for more than fifteen hundred years, it was utilised only for that purpose. It is generally agreed that Hipparchus of Nicaea, who lived around 161–126 BC, was the most accomplished astronomer of antiquity and the one who first developed trigonometry. In response to a crisis in the scientific community, trigonometry was developed. When the Greeks attempted to couch astronomy in the language of geometry, they were confronted with the unsettling reality that the heavens are lopsided. Observations of celestial events required the development of new instruments. For this reason, he developed inertial mechanics, which he constructed using the calculus tools that were still in their infancy at the time.

For this reason, we start with a stationary and unmovable earth. On top of it is the enormous dome that represents the night sky, which rotates once every twenty-four hours. In ancient times, people came to the realisation that the stars do not, in fact, disappear during the day. Despite the fact that they are present, it is hard to notice them due to the brightness of the sun. This dome does not have a predetermined location for the sun to be in. Throughout the course of the year, it moves across the constellations in a path that is known as the ecliptic, which is its own circle. Find the location of the sun in its annual voyage around this huge circle, and you will be able to determine the season by this method. The zodiac system operates in this manner. Through the identification of the constellation in which the sun is situated, the zodiac sign provides a description of the sun's position in the sky.

A small number of stars, which are referred to as wanderers or, in Greek, planetes (hence our name planets), also move across the dome of the sky while following the same ecliptic circle. The majority of stars are stationary inside the dome of the sky, which is said to be revolving. In light of the fact that the position of the sun has such a significant role in determining the seasons of heat and cold, rain and drought, it is logical to assume that the positions of the wanderers should also have significant, if more subtle, effects on our existence. Astronomy and astrology came into existence.

In the fourth century B.C., Aristotle inherited a worldview that viewed the earth as the fixed centre of the universe, with the moon, sun, and planets ensconced in concentric, ethereal spheres that spun with perfect regularity around us. This worldview proposed that the earth was the centre of the universe. In the course of over two millennia, it would serve as the foundation for a worldview that was both broad and coherent, and it would eventually endure. But it wasn't until fewer than two hundred years later that its first fractures revealed.

The First Conference on Fractional Calculus and its Applications was held at the University of New Haven in June 1974, and Bertram Ross edited the results of the conference. This event took place shortly after Ross had completed his Ph.D. dissertation on fractional calculus. The first monograph that was specifically devoted to football was released in 1974 by Keith B. Oldham and Jerome Spanier. There are already a great number of titles that are included in the series of books, journals, and conferences that are devoted to FC and its applications, and it is anticipated that this list will continue to expand in the years to come. As a result, the time has come to conduct an analysis of the progression that has taken place in this scientific field and to establish some definitive measures of this progression.

The so-called Moore's law, which is concerned with the development of computing hardware, is one of the metrics in the scientific community that is more well-known. After seeing that the number of components in integrated circuits was doubling every two years from the time of the discovery of the integrated circuit in 1958 until 1965, Gordon E. Moore came up with the idea of an exponential trendline. In light of this, he made a prediction that the pattern would persist "for at least ten years," which turned out to be correct in the years that followed. In point of fact, the trendline is utilised in the semiconductor sector for the purpose of the long-term planning as well as the definition of research and development aims. It has been observed that the rate of expansion has slowed down in recent times, and it is anticipated that the number of transistors and their density will only double every three years.

What is commonly referred to as Lotka's law, which was named after Alfred J. Lotka, is frequently seen in scientific journals. The trendline presented here provides a detailed description of the frequency of publishing by authors, highlighting the fact that the number of authors who make n contributions is about 1/nα of those who make one contribution. It was taken into consideration that the power law trendline encompasses a wide range of domains, and that the value of α is contingent upon the scientific domain, with values ranging from 2 to 3. As a matter of fact, Lotka's law is nothing more than a particular instance of the power law family of instances that describe both natural and man-made events. Quantitative research in the scientific field frequently focuses on trends in publications, such as tracking citations and co-authorship in order to evaluate the impact of collaboration and the extent of its influence. On the other hand, science metrics provides the possibility to comprehend scientific accomplishments and to have an impact on the progression of scientific knowledge.

Raymond Kurzweil is responsible for what is currently considered to be one of the most well-known speculations based on trendlines. He predicts that the human species will be able to extend their bodies and thoughts via the use of technology in the future and posits the concept of a "technological singularity" in the forthcoming years. The exponential development in the "law of accelerating returns" is something that Kurzweil notices, and he predicts what will happen to humanity in the future. The assumptions and selective use of growth metrics that Kurzweil has made have been the subject of dispute and have been vigorously questioned. The use of trendlines to depict the past is widely recognised, but it presents philosophical and conceptual challenges when it comes to forecasting the future. This discussion is not the subject of the article that is now being presented; nonetheless, it does demonstrate its acceptance. After taking these concepts into consideration, the next part will describe the development that has taken place in the field of FC. The outcomes of the proposed measurements are evaluated with the use of histograms and trendlines before being presented.

The winter solstice, the spring equinox, the summer solstice, and the autumnal equinox are the four cardinal points of the vast circle that the sun travels around. These points represent the starting and ending points of each season. In the event that the sun moves down the ecliptic at a constant pace, the length of each of the four seasons ought to be the same. This is not the case (see to Figure 1.1). There are just 89 days between the winter solstice and the spring equinox. Spring is almost ninety days away. The longest season, summer, has more than 93 and a half days. The arrival of autumn is almost 93 days away. Assuming that the sun moves at a constant pace, the only conclusion that can be drawn from this is that the earth is not centred. The challenge of determining the location of the earth was one that Hipparchus attempted to solve.

The Evolution of Fractional Calculus

In the year 1695, FC came into being as a successful continuation of Gottfried Leibniz's creative ideas. During these three centuries of growth in FC, several scientists who were significant to the field supported it, and Figure 1.1 displays a straightforward history. It is possible to mention the 1966 IBM poster titled "Men of Modern Mathematics" for general mathematics from the year 1000 AD up until the year 1950 AD. In 2012, IBM Corporation, with the assistance of the Eames Office, released an iPad application called "Minds of Modern Mathematics" that was based on the poster and updated to the present day. For more information regarding fractional calculus, readers can obtain the posters titled "Old History of Fractional Calculus" and "Recent History of Fractional Calculus" as well as the website for the Journal Fractional Calculus & Applied Analysis.

The inspection of these charts indicates, in a way that is intuitively obvious, that significant progress has been made, and the issue naturally arises as to whether or not any quantification is conceivable. To be more specific, we are discussing whether or if it is conceivable to measure the scientific activity in some way, how to construct it, and how to interpret the findings after we have obtained some quantitative data. The first assumption that has to be made is that we will use a statistical method, and that in order to accomplish this, a substantial amount of data is only accessible during the past half-century. The second presumption is that we would concentrate on publishing media that in some way reflects the knowledge that is already available. Within the context of a quantification approach, this viewpoint is equivalent to considering books and monographs to be the physical things that need to be assessed. There will be no differentiation made regarding the language, writers, amount of pages, or publisher of the work. In spite of this, we will be taking into consideration two unique approaches, namely books with authors and books edited, which will result in different sets of measures.

This tactic has both positive and negative aspects. It is possible that it will not be able to capture the entirety of scientific information owing to social or economic circumstances that restrict access to such publications, despite the fact that it offers a "counting algorithm" that is aggressive. In the field of finance, conferences that have evolved over the course of the previous few decades constitute yet another alternative collection of measuring objects. Despite this, it is challenging to differentiate between these kinds of events in terms of the number of attendees and the significance of the event itself, given that we have several types of events, such as special sessions, invited sections, workshops, conferences, and symposiums. An other alternative is to address papers that have been published in journals; however, this presents a number of challenges, including the definition of the boundaries of what constitutes FC and what does not, as well as the counting of manuscripts in a multitude of journals published by a variety of publishers. As a result, the publications signed by the author and books edited by editors that were released after 1966 were evaluated.

According to Hipparchus and his contemporaries, the fundamental issue within the field of trigonometry is as follows: Determine the length of the chord that links the endpoints of an arc provided that you are given an arc of a circle (see to Figure 1.2 for more information). There is a relationship between the length of the chord and the length of the arc as well as the radius of the circle. For the ancient Greeks, as well as for all scientists up to Newton, the number 900 did not represent the measurement of a right angle; rather, it expressed the distance that is approximately equal to one quarter of the diameter of a circle. Degrees were used as a unit of length measurement. It is reasonable to assume that the radius of a circle with a diameter of 360 degrees is equal to 360 divided by T, which is 57.2957795... With the goal of achieving higher precision, the circumference of this standard.

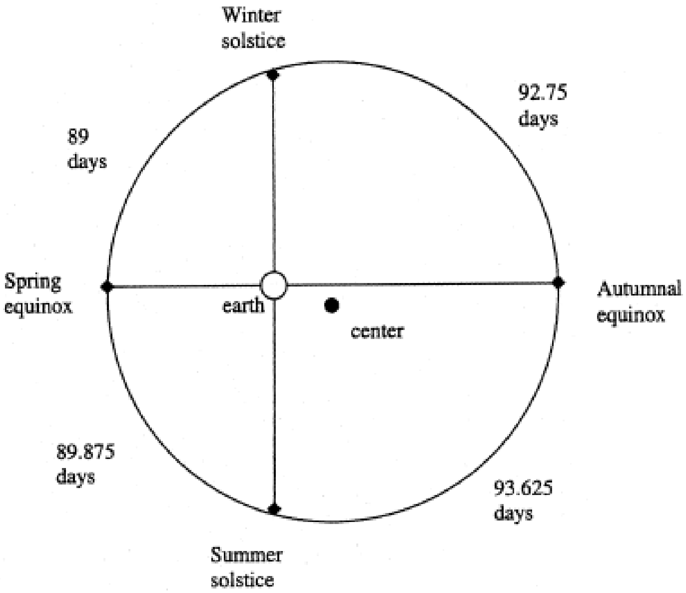

Figure 1.1 The unequal seasons, rounded to nearest 1/8 day.

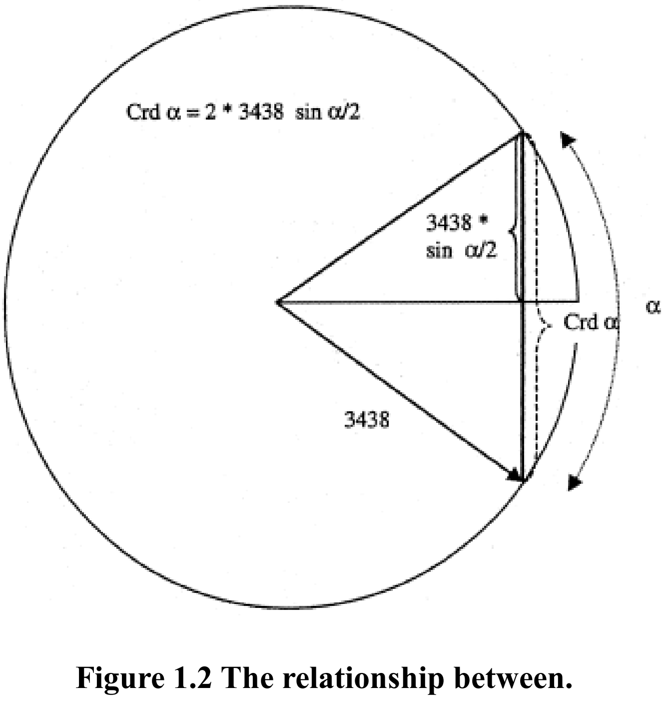

Figure 1.2 The relationship between.

One might measure the length of a circle in minutes. The radius is 3437.74677, and the circumference is 21,600 minutes, according to this calculation... When it came to Indian trigonometry, the use of a radius of 3438 would eventually become ubiquitous. It is possible that Hipparchus, whose trigonometric tables have been lost to history, also utilised a radius of 3438. There is some evidence against this possibility.

In the year 1695, Gottfried Leibniz addressed a letter to Guillaume de L'Hépital, a French mathematician, in which he posed the question, "Is it possible to generalise the meaning of derivatives with integer order to derivatives with non-integer orders?" The response that L'Hopital provided to Leibniz was to formulate a different question, which was, "What if the order will be 1 2?" The following is a response that Leibniz provided in a subsequent letter: "It will lead to a paradox, from which one day useful consequences will be drawn." A number of significant mathematicians, including Euler, Liouville, Fourier, Abel, Riemann, and Weyl, among others, contributed to the development of this theory throughout the course of more than three hundred years. The question that Leibniz presented for a fractional derivative became a matter of discussion for more than three hundred years.

One of the subfields of classical calculus, which is commonly referred to as "Fractional Calculus" (FC), is concerned with the computation of derivatives or integrals of non-integer order. The origin of the notion is reflected in this word; nevertheless, it is a misnomer; the terms "integration and differentiation of arbitrary order" or "generalised integro-differentiation" are more accurate. It wasn't until the last half-century that researchers confirmed that fractional-order models capture an innately large number of properties of both natural and artificial processes. Prior to that, fractional-order modelling was considered an abstract mathematical field.

In 1669, Newton provided a method for solving an algebraic problem, and in 1690, Raphson proposed the same method. Since then, more than three hundred years have elapsed since the two men made their respective proposals. This method, which is now known as Newton's method or the Newton–Raphson method, continues to be an important tool for solving nonlinear equations for a variety of reasons. Numerous subjects that are associated with Newton's technique continue to garner the interest of researchers.

As an illustration, the development of globally convergent and effective iterative approaches for the resolution of non-convergent problems rentable equations in $\mathbb{R}^n$ or $\mathbb{C}^n$ As an illustration, the development of globally convergent and effective iterative approaches for the resolution of non-convergent problems.

Error estimates for Newton's technique and Newton-like methods, mostly for differentiable equations in Banach spaces, are the focus of this investigation, which aims to track historical advancements in Kantorovich-type convergence theory. Additionally, the study will evaluate the accuracy of these approaches.

In addition to providing error estimates for Newton's technique that are based on the Newton–Kantorovich theorem, we present essential discoveries that have been discovered throughout the history of convergence investigation. We declare certain convergence findings for Newton-like techniques, which are generalisations of the results for Newton's method.

Let X and Y occupy places in the Banach $F : D \subseteq X \to Y$ operate in a situation where D holds a place in the $F.$ if $F.$ in a set that is open and convex, is distinguishable $\overline{D_0} \subseteq D,$ Thereafter, the problem may be solved using Newton's technique.

$$F(x) = 0 \quad \textbf{(1.1)}$$

To provide a solution x^* the following serves as the defense:

(N1) Let us x_k approximately correspond to the x^*;

(N2) Resolve the equation that is linear.

$$F(x_k) + F'(x_k)h = 0 \tag{1.2}$$

and with regard to the letter h, given that $F'(x_k)$ has no single form;

If a convergence theorem for an iterative technique claims convergence by assuming that a solution exists, then the theorem is referred to as a local convergence theorem. x^* an initial estimate is selected that is sufficiently near to the value that exists x^* On the other hand, a semilocal convergence theorem is a type of convergence theorem that does not presuppose the existence of any solution a priori, but rather asserts that certain conditions apply at the beginning point x0. This type of theorem is similar to Theorem 2.1.

Ostrowski increased the accuracy of Theorem 2.1 by substituting (2.5) with $2K\eta \leqslant |f'(x_0)|$ as well as demonstrating.

$$|x_{k+1} - x_k| \leqslant \frac{K}{2|f'(x_0)|}|x_k - x_{k-1}|^2, \quad k \geqslant 1. \tag{1.3}$$

Newton-like methods

Because of its strength, the majorant principle, which was developed by Kantorovich, has been utilised by a multitude of writers in order to derive convergence theorems for various variations of Newton's method. In particular, Rheinboldt utilised his majorant theory in order to get a convergence theorem for the Newtonian method.

$$x_{k+1} = x_k - A(x_k)^{-1}F(x_k), \quad k = 0, 1, 2, \ldots, \tag{1.4}$$

in the Banach space, for the purpose of solving Equation (2.1), where $F: D \subseteq X \to Y$ exists in a convex set that is open and differentiable $D_0 \subseteq D$, $x_0 \in D_0$ and $A(x)$ represents a linear operator that is invertible and bounded, and it may be thought of as an approximation to $F'(x)$.

CONCLUSION

Permit me to start by providing some context for this problem. The presumption that the earth is stationary is the starting point for this one. This was a topic of discussion in the early Greek scientific community: does the sun revolve around the earth, or does the earth revolve around the sun?There is a strong indicator that the earth does not move, and that is the fact that we do not feel any feeling of motion. In point of fact, when it became apparent in the early seventeenth century that the earth revolved around the sun, it posed a significant challenge for scientists: how could they explain how this was even possible? If we were travelling through space at much higher speeds and whirling at thousands of miles per hour, how could it be that we were not feeling any of these things? There is no doubt that if the earth were to shift, we would have been thrown off a long time ago. Newton's most significant achievement in the Principia was to find a solution to this dilemma.

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